

VILNIUS GEDIMINAS TECHNICAL UNIVERSITY

Rasa ŽYGIENĖ

RESEARCH ON THE DYNAMIC  
PROCESSES OF THE INTERACTION  
BETWEEN THE DAMAGED WHEELS  
OF A RAILWAY VEHICLE AND RAILS

SUMMARY OF DOCTORAL DISSERTATION

TECHNOLOGICAL SCIENCES,  
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Scientific Supervisor

**Prof Dr Habil Marijonas BOGDEVICHIUS** (Vilnius Gediminas Technical University, Technological Sciences, Transport Engineering – 03T).

**The dissertation is being defended at the Council of Scientific Field of Transport Engineering at Vilnius Gediminas Technical University:**

Chairman

**Assoc Prof Dr Olegas PRENTKOVSKIS** (Vilnius Gediminas Technical University, Technological Sciences, Transport Engineering – 03T).

Members:

**Prof Dr Habil Vytautas BARZDAITIS** (Kaunas University of Technology, Technological Sciences, Mechanical Engineering – 09T),

**Assoc Prof Dr Algirdas JANULEVICIUS** (Aleksandras Stulginskis University, Technological Sciences, Transport Engineering – 03T),

**Prof Dr Habil Henrikas SIVILEVICIUS** (Vilnius Gediminas Technical University, Technological Sciences, Transport Engineering – 03T),

**Assoc Prof Dr Edgar SOKOLOVSKIJ** (Vilnius Gediminas Technical University, Technological Sciences, Transport Engineering – 03T).

Opponents:

**Prof Dr Žilvinas BAZARAS** (Kaunas University of Technology, Technological Sciences, Transport Engineering – 03T),

**Assoc Prof Dr Gintautas BUREIKA** (Vilnius Gediminas Technical University, Technological Sciences, Transport Engineering – 03T).

The dissertation will be defended at the public meeting of the Council of Scientific Field of Transport Engineering in the Senate Hall of Vilnius Gediminas Technical University at **2 p. m. on 22 June 2015**.

Address: Saulėtekio al. 11, LT-10223 Vilnius, Lithuania.

Tel.: +370 5 274 4952, +370 5 274 4956; fax +370 5 270 0112;

e-mail: doktor@vgtu.lt

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VILNIAUS GEDIMINO TECHNIKOS UNIVERSITETAS

Rasa ŽYGIENĖ

# GELEŽINKELIO RIEDMENŲ RATŲ SU PAŽAIDOMIS IR BĖGIŲ SĄVEIKOS DINAMINIŲ PROCESŲ TYRIMAS

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Mokslinis vadovas

**prof. habil. dr. Marijonas BOGDEVIČIUS** (Vilniaus Gedimino technikos universitetas, technologijos mokslai, transporto inžinerija – 03T).

**Disertacija ginama Vilniaus Gedimino technikos universiteto Transporto inžinerijos mokslo krypties taryboje:**

Pirmininkas

**doc. dr. Olegas PRENTKOVSKIS** (Vilniaus Gedimino technikos universitetas, technologijos mokslai, transporto inžinerija – 03T).

Nariai:

**prof. habil. dr. Vytautas BARZDAITIS** (Kauno technologijos universitetas, technologijos mokslai, mechanikos inžinerija – 09T),

**doc. dr. Algirdas JANULEVIČIUS** (Aleksandro Stulginskio universitetas, technologijos mokslai, transporto inžinerija – 03T),

**prof. habil. dr. Henrikas SIVILEVIČIUS** (Vilniaus Gedimino technikos universitetas, technologijos mokslai, transporto inžinerija – 03T),

**doc. dr. Edgar SOKOLOVSKIJ** (Vilniaus Gedimino technikos universitetas, technologijos mokslai, transporto inžinerija – 03T).

Oponentai:

**prof. dr. Žilvinas BAZARAS** (Kauno technologijos universitetas, technologijos mokslai, transporto inžinerija – 03T),

**doc. dr. Gintautas BUREIKA** (Vilniaus Gedimino technikos universitetas, technologijos mokslai, transporto inžinerija – 03T).

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Adresas: Saulėtekio al. 11, LT-10223 Vilnius, Lietuva.

Tel.: (8 5) 274 4952, (8 5) 274 4956; faksas (8 5) 270 0112;

el. paštas doktor@vgtu.lt

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## Introduction

**Problem formulation.** Railway traffic safety, stability, noise and comfort depend on the interaction between a wheel and a rail. The problem of the interaction between the wheel and the rail, taking into account an increase in loads and velocities, is becoming incredibly important.

The distribution of forces occurring in the interaction between the wheel and the rail has an influence on the dynamics of a vehicle and a track. Contact forces are directly dependent on the velocity of the railway vehicle in motion, the wheel, the rail and their damages, geometrical parameters and the physical and mechanical properties of the system “Vehicle-Track”.

In order to reduce the wear of the wheel and the rail, to prolong the operational lifetime of deterioration and to protect the environment, the processes of the interaction between the wheel and the rail must be researched thus solving the compatibility problems of the surfaces of the wheel and rail structure as well as monitoring and predicting there condition.

The main problem, in terms of traffic safety, is the rolling stability of the vehicle and derailment. The wheel may lose its contact with the rail, the vehicle may roll off the rails and result in an accident or a disaster, which may cause damage to the environment, human health and life due to the fact that wheel flats and impact forces occur in the wheel-rail contact.

As regards mathematical models for the interaction between the wheel and the rail, losses of the wheel-rail contact, possible gaps between the rail and the sleeper or ballast are often underestimated or only a part of these conditions are assessed.

Therefore, it is appropriate to create a mathematical model for the interaction between the wheel and the rail, which allows describing the geometry of the wheel flat and the wheel itself more accurately and evaluating a possible loss of the wheel-rail contact and gaps between the rail and the sleeper, the sleeper and ballast.

Complex mathematical models and computer software are used in order to determine maximum impact forces regarding the interaction between the wheel and the rail. When the railway wheel has a flat, lack of scientific solutions to the simplified calculation methods of maximum force can be observed.

**Relevance of the thesis.** Different size forces occur in the interaction between the wheel and the rail due to the structures of the wheel and the rail, rail joints, welded joints, different velocities of the vehicles, braking forces, wheel blocking and vehicle mass. These forces cause friction between the wheel and the rail, faster depreciation of their working surfaces and may reduce their operational time.

A number of authors describe the contact zone of the wheel and the rail as the point and the geometry of the wheel as an analytical function in the mathematical models for the interaction between the wheel flat and the rail. There is no single method accurately determining dynamical processes occurring in the contact zone of the wheel and the rail.

To ensure safe traffic and environment, human health and even life, the interaction between the damaged wheel and the rail must be taken into account as a serious problem. It is appropriate to create a simplified methodology determining the parameters of the processes occurring in contact between the wheel and the rail to ensure a longer operational life of the vehicle and the rail.

**Object of research.** The object of the conducted research is the interaction between the damaged wheel of the vehicle and the uneven rail.

**Aim of the thesis.** The thesis is aimed at developing a calculation method for the system “Vehicle-Track” allowing the determination of dynamic loads occurring within the interaction between the damaged wheel and the uneven rail, their dependencies on the geometrical parameters of the system and vehicle velocity.

**The objectives of the thesis.** To achieve the purpose of the thesis, the following tasks have been formulated:

1. Reviewing research methods for the interaction between the wheel of the vehicle and the rail and analysing damage to the wheel and the rail.
2. Establishing a mathematical model for the system “Vehicle-Track” to evaluate damage to the wheel and rail unevenness.
3. Creating a simplified methodology allowing the determination of the influence of flat length and vehicle velocity on maximum forces acting in the contact area of the wheel and the rail.
4. Comparing research results obtained using the developed methodology for determining vertical contact load (KAM) with research results received by other authors and ATLAS-LG data obtained under real operational conditions.

**Research methodology.** The thesis applies to research based on methodologies employed in works done by Lithuanian and foreign scientists. In order to create the simplified methodology for determining vertical contact force and the system “Vehicle-Track”, analytical, statistical and numerical research methods have been applied.

ATLAS-LG has been used for evaluating forces in the wheel-rail contact. Special RDM-22s software employed in the defectoscope for analysing rail flats has been applied.

Mathematical modelling has been carried out in the interface of the “Visual Studio” environment. To process results, “Fortran”, “Maple”, “Matlab”, “R” and “Microsoft Office” software have been used.

***Scientific novelty of the thesis***

1. Geometrical parameters of the wheel, the rail and their damage and the influence of vehicle masses and gaps forming between the rolling surfaces of the wheel and the rail on the parameters must be evaluated when using the mathematical model of the system “Vehicle-Track” for determination of dynamical processes.
2. The simplified methodology for determining a vertical contact load has been created to describe contact characteristics and generated forces by implicating numerical and experimental methods. The forces occurring within contact are identified using the methodology for determining a simplified contact load under different parameters.
3. A model for the system “Vehicle-Track” has been designed in the Visual Studio environment, which allows defining sliding velocity, longitudinal elastic slips, friction and normal forces and the torque of friction forces at any time. With reference to this data, a possibility of establishing the average values of these parameters in the contact zone of the wheel and the rail at any time arises.

***The practical of the research findings.*** The paper presents the mathematical model for the damaged vehicle wheel of the system “Vehicle-Track”. The model defines the forces occurring due to the interaction between the damaged wheel and the rail as well as evaluates vehicle velocity, the geometrical parameters of the interacting bodies and their physical and mechanical properties. The developed mathematical model allows determining changes in forces in contact.

With the help of the mathematical model designed for the system “Vehicle-Track” in the wheel-rail contact area between two contact points of the surface, the springy sliding friction of the wheel-set, friction forces, torque moments acting on the wheel, split load and contact forces are determined at each time point in order to lengthen the operational life of the wheel and the rail, reduce train accidents and protect the environment, human health and life.

The methodology for establishing vertical contact loads has been designed examining the impact of the geometrical parameters of damage to the wheel and the rail on the dynamic processes of the system “Vehicle-Track”. Damage to wheels can be identified using maximum force values obtained from the ATLAS-LG subsystem and the forces received applying the simplified methodology for vertical contact forces.

### ***Defended statements***

1. The model of the system "Vehicle-Track" evaluates the geometrical parameters of the wheel, the rail and their damage and assesses the influence of vehicle masses and gaps forming between the rolling surfaces of the wheel and the rail on the parameters of the analysed system.
2. In order to evaluate contact forces and dynamical processes, including sliding velocities, longitudinal elastic slips, normal and frictional forces and the torque of frictional force acting on the wheel-set and other processes of the system "Vehicle-Track" at every time moment, the contact area of the wheel and the rail is divided into a desired number of intervals, in which, damages in wheel and rail and micro unevenness are calculated.
3. It is possible to identify maximum contact forces occurring in contact between the damaged wheel and rail, by evaluating velocity, wheel and rail geometrical parameters and the angle of deviation, and by creating the simplified methodology for determination of vertical contact loads.
4. It is possible to determine the distribution of impact force and its changes in time, using derived impact force dependencies.

***Structure of the dissertation.*** The dissertation consists of the introduction, four chapters, a list of references, a list of author publications and four annexes. The volume of the paper is 116 pages (excluding annexes), including 123 equations, 52 figures and 9 tables. 108 references have been used in the dissertation.

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## 1. A review of scientific works examining the interaction between the wheel and the rail and the analysis of defects in the rail and wheels

The main problem of railway transport is the interaction between the wheel and the rail, as these are the aspects that make an impact on traffic safety. The dynamical interaction of the wheel and the rail induces impact forces that may occur due to damage to the wheel and the rail, rail joints, etc., which influences railway traffic. Any kind of interference in the railway track may cause considerable damage to a railway company.

The conducted research has referred to the interaction between the wheel and the rail and to the main evaluation theories of the interaction between the wheel and the rail, including those of Carter, Hertz, Johnson – Kendall – Roberts (JKR), Kalker, Braddley, Winkler, etc. J. J. Kalker widely describes Carter, Hertzian and JKR theories, their capabilities, limitations and application. The Carter theory describes the actions of wheel acceleration and braking when high tangential forces are transmitted to the rail that is considered as an infinite beam. Vehicle transverse forces are underestimated when using the Carter method. The most popular one is the Hertzian theory of elastic bodies stating that when two oval bodies are squeezed, the contact area is elliptical and maximum pressure forms in the centre of it. It is possible to determine the maximum load after the integration of pressure distribution in the contact area.

A number of authors differently evaluate the depth of the flat. The calculation of the geometry of the wheel flat of the vehicle by the authors is as follows:

- Pieringer, Wu, Steenberg,  $\Delta_F = L_F^2 / 8R_W$  ;
- Zhu,  $\Delta_F = L_F^2 / 16R_W$  ;
- When the flat is a cord (Zhu),  $\Delta_F = R_W - \sqrt{R_W^2 - (L_F^2 / 4)}$  ;
- Sladkovskij,  $L_F = 2\sqrt{2R_W\Delta_F - \Delta_F^2}$  ;
- Uzzal,  $R_W = 1/2 (\Delta_F) [1 - \cos (2\pi x / L_F)]$  ;

where  $\Delta_F$  – depth;  $L_F$  – length,  $x$  – a longitudinal coordinate of the flat at the contact point.

Different diameters of the wheel have an impact on the depth of the flat, which influences flat length.

Dynamic forces occur in the interaction between the wheel and the rail, in case of wheel flats, which loads the wheel and the rail. Exposure time for dynamical forces occurring in the contact area of the wheel and the rail depends on vehicle velocity and flat length.

Complex calculations are carried out for examining the interaction between the wheel and the rail and complex computer programs are created, because the parameters of the system “Vehicle-Track” are influenced by various factors.

Many mathematical models do not evaluate the possibility of contact loss in the interaction of the wheel and the rail, the rail and the roadbed or assess only conditions.

Numerous studies have shown the significance of this area and the need to solve the problems of the interaction between the wheel and the rail.

A review of scientific works, under the topic of the interaction between the wheel and the rail, has shown that the dynamics of the vehicle is highly influenced by the unevenness of the rail with short waves.

The state of the rail is mostly affected by the form of the rolling surface of the vehicle wheel.

A wheel flat is the most common damage to the vehicle wheel and the one used by many authors in the models for the interaction between the wheel and the rail due to the simplicity of calculations. The authors differently describe the geometry of wheel flats.

## **2. Mathematical modelling of the system “Vehicle-Track”**

The developed mathematical model for the system “Vehicle-Track” can be used for determining the interaction between the vehicle wheel and the rail when the wheel has or has no flat.

For examining the interaction of the elements in the system “Vehicle-Track”, the following assumptions and evaluation have been made:

1. Rail deformation in the  $X, Z$  plane.
2. Rail interaction with the roadbed as a tough base.
3. Potential space between the rail and the roadbed is possible.
4. The impact of axial forces in the rail on stiffness (due to difference in temperatures).
5. The initial deformation of the rail.
6. A gap between the sleeper and the rail.
7. A gap between the rail and the roadbed in the middle of two sleepers.
8. Damaged wheel profile.

The estimated vehicle in the dynamic model for the system “Vehicle-Track” consists of 1/8 of the car body with cargo mass ( $m_{bg4}$ ), 1/4 of bogie mass ( $m_{bg3}$ ) and 1/2 of wheel-set mass ( $m_{bg2}$ ).

The mass of the wheel and the wheel-set is divided into two parts: wheel mass  $m_{bg1}$  that is in direct contact with the rail and the main mass of the wheel-set  $m_{bg2}$ . The use of wheel mass  $m_{bg1}$ , which is directly in contact with the rail, allows a more accurate assessment of forces acting in the interaction of the wheel and the rail and of the kinetic parameters of individual wheel parts.

The track model for the system “Vehicle–Track” consists of a rail sleeper ( $m_{sl}$ ) and a roadbed. The roadbed consists of ballast, sub-ballast and soil layers ( $m_{s1}, m_{s2}, m_{s3}$ ).

As for the mathematical model for the rail and the wheel, micro unevenness in the contact length of the wheel and the rail, local slips, normal and tangential forces and the moments of forces are evaluated.

Contact length is divided into small parts in the mathematical model, which assists with evaluating forces according to the Hertzian contact theory. The second row Hermite polynomials is used for describing rail unevenness.

The depth of the wheel flat is described as

$$\Delta_F = R_{W0} \left( 1 - \sqrt{1 - \left( \frac{L_F}{2R_{W0}} \right)^2} \right) \quad (\text{S.2.1})$$

where  $R_{W0}$  – the nominal radius of the wheel;  $L_F$  – the length of the wheel flat.

The penetration rate at point  $k$  is:

$$\begin{aligned} \dot{\delta}_k = & [N_w(\xi_k)] \{\dot{q}_e\} + \frac{V}{L_{Re}} \left[ \frac{\partial N_w(\xi_k)}{\partial \xi} \right] \{q_e\} + \\ & + \frac{V}{L_{Re}} \frac{d\Delta Z}{d\xi} \cos(\varphi_k) - \Delta Z \dot{\varphi}_k \sin(\varphi_k) - \dot{q}_{bg1} - \frac{\partial R_k(\theta_k)}{\partial \theta} \Omega \sin(\Psi_k), \end{aligned} \quad (\text{S.2.2})$$

where  $[N_w(\xi_k)]$  – the matrix of the finite element-shaped function;  $L_{Re}$  – matrix length of the rail;  $q_{bg1}$  – vertical displacements of the wheel;  $\Delta Z$  – vertical unevenness of the rail;  $\{q_e\}$  – the displacement vector of the finite element;

$\dot{\varphi}_k = \cos^2(\varphi_k) \left( \frac{1}{L_{Re}} \left[ \frac{\partial N_w(\xi_k)}{\partial \xi} \right] \{\dot{q}_e\} + \frac{V}{L_{Re}^2} \left[ \frac{\partial^2 N_w(\xi_k)}{\partial \xi^2} \right] \{q_e\} \right)$ ;  $\{\dot{q}_e\}$  – the velocity vector of finite elements;  $\Psi_k$  – angle;  $\Omega$  – the angular velocity of the wheel,  $\Omega = V/R_{W0}$ .

Force  $F_{contact}(x)$  at the contact point is determined using the Hertzian theory:

$$F_{contact}(x) = k_{RW}(x) \delta(x)^n D(\dot{\delta}(x)) H(\delta(x)), \quad (S.2.3)$$

where  $e_C$  – velocity recovery rate;  $n$  – exponent;  $H$  – Heaviside function;  $\delta$  – penetration;  $D(\dot{\delta}) = 1 + \frac{3}{4} \left(1 - e_C^2\right) \frac{\dot{\delta}}{\dot{\delta}_{max}}$ ;  $k_{RW}$  – the coefficient of contact

$$\text{stiffness, } k_{RW}(x) = \frac{4}{3} E_{ekv} \sqrt{R(x)}.$$

A system of nonlinear equations for the movement of the vehicle out of contact is equal to

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} + \{F_{NL}(q, \dot{q})\} = \{F(t)\}, \quad (S.2.4)$$

where  $[M]$ ,  $[C]$ ,  $[K]$ ,  $\{F_{NL}(q, \dot{q})\}$ ,  $\{F(t)\}$  – mass, damping and stiffness matrices, nonlinear load and weight force load vectors respectively;  $\{q\}^T = [\{q_R\}^T, \{q_B\}^T, \{q_{bg}\}^T]$  – a sum of displacement vectors;  $\{q_R\}$  – a vector of rail displacements;  $\{q_S\}$  – a vector of soil displacements,  $\{q_{bg}\}$  – a vector of vehicle displacements.

Non-linear force  $\{F_{NL}(q, \dot{q})\}$  is extracted in the Taylor series at the surroundings of point  $\{q_k\}$ :

$$\{F_{NL}(q, \dot{q})\} = \{F_{NL,k}\} + [K_{T,k}]\{\Delta q_k\} + [C_{T,k}]\{\Delta \dot{q}_k\}, \quad (S.2.5)$$

$$\text{where } [K_{T,k}] = \left[ \frac{\partial \{F_{NL}(q_k, \dot{q}_k)\}}{\partial \{q\}} \right], \quad [C_{T,k}] = \left[ \frac{\partial \{F_{NL}(q_k, \dot{q}_k)\}}{\partial \{\dot{q}\}} \right].$$

Then, the total system for equations, at the moment of time  $t + \Delta t$ , is equal to

$$\begin{aligned} & [M]\{\ddot{q}_{t+\Delta t}\} + [C]\{\dot{q}_{t+\Delta t}\} + [K]\{q_{t+\Delta t}\} + [C_T]\{\Delta \dot{q}_{t+\Delta t,k}\} + \\ & + [K_T]\{\Delta q_{t+\Delta t,k}\} = -\{F_{NL}(q_{t+\Delta t}, \dot{q}_{t+\Delta t})\} + \{F(t)\}. \end{aligned} \quad (S.2.6)$$

Each time, the mathematical model of the damaged wheel and uneven rail for the system “Vehicle-Track” allows determining stiffness between the wheel and the rail, sliding speeds, longitudinal elastic slip, friction and normal forces,

the torque of friction forces acting on the wheel and the average values of these characteristics at each contact point.

### 3. The methodology for determining a vertical contact load in the wheel

When the railway vehicle reaches a certain velocity, force  $\{F(t)\}$  acts in the interaction between the wheel and the rail. It can be divided into contact  $\{F_{stat}(t)\}$  and impact  $\{F_{dinam}(t)\}$  forces:

$$\{F(t)\} = \{F_{stat}\} + \{F_{dinam}(t)\} . \quad (S.3.1)$$

When a wheel flat of the railway vehicle moves at a certain speed, a short-term loss of wheel-rail interaction is possible due to impact force  $F_{dinam}(t)$  emerging in the interaction, and force during contact is equal to zero,  $F(t) = 0$ .

When determining the impact force of wheel–rail interaction, the following assumptions and evaluation can be made:

1. Velocities of the interacting bodies are perpendicular to contact surfaces.
2. The distribution of contact forces depends upon velocity distribution in the contact area.
3. In accordance with the distribution of velocity, the contact area is divided into two zones.
4. The distribution of forces is different in each contact area.
5. Mechanical power inputs are proportional in every contact area (parameter  $\lambda$ ).
6. Maximum dynamic contact force acts in the geometric centre of velocity distribution.
7. The angle (deviation angle,  $\Psi$ ) between rail deflection and the horizontal axis.
8. Time for interaction is proportional to the length  $L_F$  of flats.

According to the KAM methodology, the force expression is obtained:

$$\begin{aligned} F_{dinam}(\xi) = & a_1(L_0\xi)^{n_1} [H(\xi) - H(\xi_C)] + \\ & + a_2L_0^{n_2}(1-\xi)^{n_2} [H(\xi - \xi_C) - H(\xi - 1)] , \end{aligned} \quad (S.3.2)$$

where  $H(\xi)$  – Heaviside function;  $a_1, a_2, n_1, n_2$  – unknown parameters of the mathematical model;  $L_0$  – length in the contact area;  $\xi$  – a non-dimensional coordinate,  $\xi = (X - X_0)/L_0$ .

When flat depth is lower than the maximum geometrical one, the maximum force is equal to:

$$F_{dinam \Delta} = F_{dinam} \left( \frac{\Delta_F}{\Delta_{F_{dinam}}} \right). \quad (S.3.3)$$

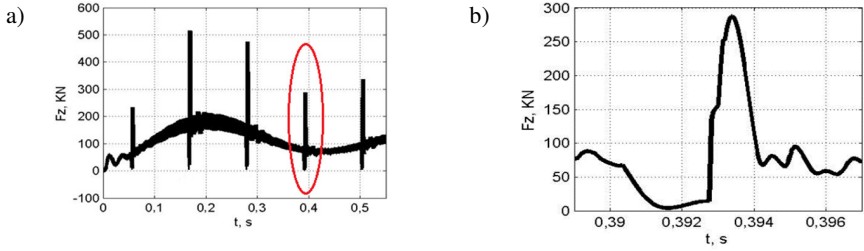
The novelty of the method determining a vertical contact load is that the length of the wheel-rail contact, according to the distribution of velocities, is divided into two zones, and the distribution of impact force determined in each zone and mathematical dependencies of contact-impact force on the parameters of the system “Vehicle-Track” are obtained and include velocity, flat dimensions, the mass and velocities of the contacting body, rail deflection and the geometrical parameters of the wheel.

#### **4. The results of mathematical modelling and the practical application of the methodology for determining a vertical load on the wheel**

The initial length of the vehicle flat is  $L_F = 100$  mm and depth is  $\Delta F = 2.53$  mm. A profile of the wheel flat of the vehicle is described using a four hundred and one harmonics ( $NH = 401$ ).

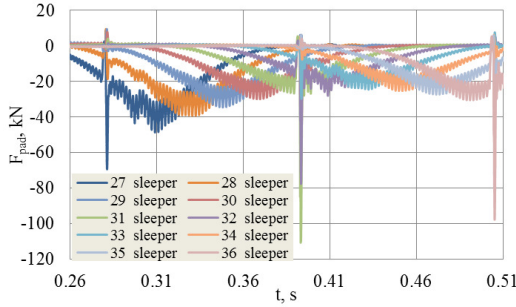
Rail R65, the length of which is 16,2 m, has been examined. The distance between sleepers is 0.5435 m. The number of points in the contact interval is equal to  $NP = 1001$ . The total time of vehicle motion is 0.560 s and integration time step is  $\Delta t = 10^{-6}$  s. The total number of unknowns is 970.

Changes in vertical force  $F_Z$  operating upon the wheel over time are shown in Fig. S.4.1. a. Wheel-rail loses its contact five times in 0.56 s. The force expanded at the time from 0.39 s to 0.397 s is shown in Fig. S.4.1. b, where force  $F_Z = 0$  shows the loss of the wheel-rail contact, and the length of the flat is  $L_F = 100$  mm, the depth of the flat is  $\Delta_F = 2.53$  mm, wheel load is 100 kN, vehicle velocity is  $V = 100$  km/h at the time step from 0.3914 s to 0.3917 s. The loss of the wheel-rail contact occurs at a very short moment of time that is equal to 0.3 ms.



**Fig. S.4.1.** Changes in vertical force operating upon the wheel over time:  
a) to 0.55 s; b) from 0.388 to 0.398 s

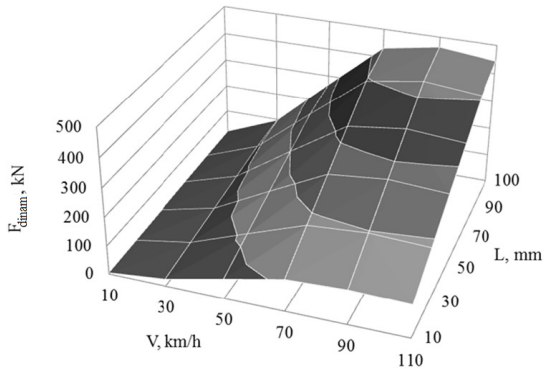
In order to determine whether the vehicle wheel has flats, forces operating on sleepers when the wheel passes a certain section of railway rails have to be identified. Forces operating from the rail (through the pad), when the wheel flat ( $L_F = 100$  mm,  $\Delta_F = 2.53$  mm) is moving under a velocity of 100 km/h, are shown in Fig. S.4.2.



**Fig. S.4.2.** Forces operating from the rail (through the pad)

Figure S.4.2 shows that an impact on the rail occurs between the 31st and the 32nd, id est., at a distance of 0.2065 m from the centre of the 31st sleeper. Impact force on the rail affects four sleepers, and both sides obtain the most significantly affected sleeper (31st sleeper).

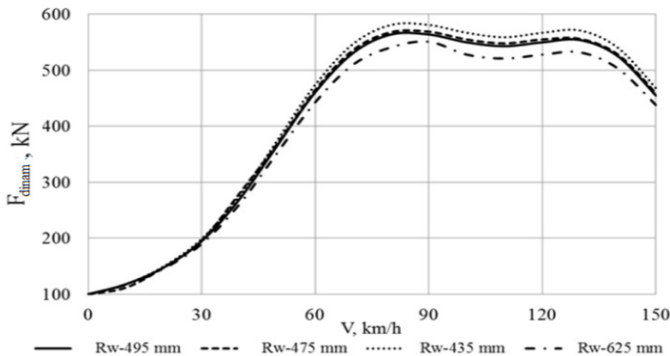
The dependency of maximum impact force  $F_{dynam}$  on the most affected parameters of the system is determined according to the developed method (Fig. S.4.3.). While calculating impact force  $F_{dynam}$ , it is assumed that the vertical load of the wheel on the rail is 100 kN and impact force between wheel flats and the rail acts in the middle of the rail located between two sleepers.



**Fig. S.4.3.** The dependency of impact force  $F_{dinam}$  on the velocity of the vehicle and the lengths of wheel flat

It has been found that if velocity  $V$  of the vehicle and length  $L$  of the wheel flat increase, impact force  $F_{dinam}$  also increases.

The dependency of impact force  $F_{dinam}$  on the velocity of vehicle  $V$  and different lengths of wheel radius, when the length of the wheel flat is  $L_F = 100$  mm, is presented in Fig. S.4.4. For a comparison of impact forces, the radii of vehicle wheels and locomotive 2TE121 (2ТЭ121) are used.



**Fig. S.4.4.** The dependency of impact force  $F_{dinam}$  on vehicle velocity  $V$  and radii  $R_W$  of wheel with flats ( $L_F = 100$  mm,  $\Delta_F = 2.53$  mm)



Impact force that appears in contact between the wheel flat and the rail and obtained using the developed method is compared with adaptive and Hertzian contact methods and the results received by Nielson. The results of variations in maximum force obtained using the KAM method when the vehicle is moving at different velocities, the wheel radius is  $R_W = 0.475$  m, flat depth is 1.56 mm, its length is 100 mm and a static load of the wheel on the rail is unvalued have been compared with the findings obtained using Zhu and Hertzian contact methods. A comparative analysis of KAM with contact force methods used by other authors has disclosed that, under the length of the wheel flat of 100 mm and its depth of 1.56 mm, the maximum values of forces up to 40 km/h are closer to the Hertzian method and starting from 70 km/h – to the Zhu contact method.

The KAM method is compared with Nielson linear, non-linear and Zhu methods when the depth of the flat is 0.9 mm, and its length is 100 mm. When flat depth is 0.9 mm, maximum force values, under the vehicle velocity of 43 km/h, are closer to the Nielson linear method while when velocity is from 60 km/h to 100 km/h – the KAM method reaches top values. The KAM values of maximum forces coincide with the methods suggested by other authors when velocity is in the range from 43 km/h to 56 km/h.

## General conclusions

1. The mathematical model for the system “Vehicle-Track” based on the split of the contact area and evaluating damage to the wheel, rail roughness and penetration has been developed. Each time, the mathematical model for the contacting surfaces between two points enables the determination of elastic slip speeds, longitudinal slip, friction and normal forces, friction forces affecting the wheel-set and the average values of these characteristics in order to extend the service life of the wheel and the rail, reduce train traffic accidents, protect the environment, human health and life.
2. The use of the mathematical model for the developed system “Vehicle-Track” has assisted in examining the dynamic processes of the system “Vehicle-Track” under the vehicle speed of 100 km/h, the static axial load 100 kN of the wheel,  $R_{W0} = 0.495$  m,  $\Delta_F = 2.53$  mm and  $L_F = 100$  mm. It has been found that impact force loads the rail on four sleepers on both sides starting from the most loaded sleeper. A short-term loss of contact is observed during the comparison of vertical displacements of wheel flats and the rail. Vertical wheel flats, rail contact force and acceleration spectra of the wheel-set for the system

“Vehicle-Track” have been obtained. The maximum amplitude of vertical wheel-rail force (40 kN) is observed at  $f = 503$  Hz. The maximum value of the acceleration amplitude is monitored at  $2f = 1006$  Hz.

3. Research on dynamic processes in the system “Vehicle-Track” has been made, when wheel flats differ in sizes, move at different velocities and carry various static loads on the wheel. It has been established that the higher is vehicle speed and the lower is the vertical load of the wheel, the larger are the displacement amplitude of mass  $m_{bg1}$  and frequency variation. Impact force has been found increases when the radius of the wheel decreases and the angle of deviation  $\Psi$  increases, and when the coefficient of proportionality  $\lambda$  increases, maximum impact force decreases.
4. The obtained value of the simplified method for determining a vertical contact load has been compared with the system ATLAS-LG and contact force determination methods used by other authors. The proposed methods have been discovered to coincide with the values obtained employing other methods under the movement velocity of up to 56 km/h. Vehicle contact forces occurring on the rail have been examined for the ATLAS-LG system at this speed.
5. Statistical calculations have disclosed, that, according to the results of the simplified methodology for determining vertical contact loads obtained using the ATLAS-LG system, the variance of relative deviations vary negligible and both calculation methodologies are equally reliable.

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### **About the author**

Rasa Žygienė (Musteikytė) was born on 1973 01 03 in Kaunas, where she has graduated Kaunas S. Darius and S. Girėnas high school. Then in 2002 she has graduated Vilnius Petras Vileišis further education transport school, where she has acquired qualification of railway traffic administrator. In 2007 at Vilnius Gediminas technical university, Rasa has acquired transport engineering high education diploma. In 2009 at the same university she has acquired master degree in transport engineering. From 2009 to 2015 she has been a PhD student in Vilnius Gediminas technical university. From 1999 to 2013 she worked at “Lietuvos geležinkeliai” corporation.

## **GELEŽINKELIO RIEDMENŲ RATŲ SU PAŽAIDOMIS IR BĖGIŲ SĄVEIKOS DINAMINIŲ PROCESŲ TYRIMAS**

### **Įvadas**

**Problemos formulavimas.** Geležinkelių eismo sauga, stabilumas, triukšmas ir komfortas priklauso nuo rato ir bėgio sąveikos. Rato ir bėgio sąveikos uždavinys didėjant apkrovoms ir greičiams tampa vis aktualesnis.

Rato ir bėgio kontakte atsirandančių jėgų pasiskirstymas turi įtakos riedmens ir kelio dinamikai. Kontaktinių jėgų dydis tiesiogiai priklauso nuo riedmens judėjimo greičio, rato ir bėgio bei jų pažaidų (defektų) geometrinių parametrų ir sistemos „Riedmuo-Kelias“ fizinių ir mechaninių savybių.

Riedėdamas bėgiu riedmens ratas dėl bėgio profilio, rato paviršiaus geometrinių netikslumų bei pažaidų sukelia dinامينius poslinkius, smūgius, vibra-

cijas, triukšmą. Triukšmas ir vibracijos neigiamai veikia aplinką, trukdo žmonių darbui, poilsui, neigiamai veikia jų sveikatą (Mačiūnas *et al.* 1999).

Ratų ir bėgių pažaidos (defektai) retai aptinkamos pradinėse eksploatacijos stadijose, o laikui bėgant jos didėja.

Norint sumažinti rato ir bėgio nusidėvėjimą ir pailginti jo eksploatacinį laikotarpį bei saugoti aplinką reikia tyrinėti rato ir bėgio sąveikos procesus, spręsti rato ir bėgio konstrukcijos paviršių suderinamumo uždavinius bei kontroliuoti ir prognozuoti jų būsenas.

Pagrindinė problema eismo saugos požiūriu yra geležinkelio riedmenų riedėjimo stabilumas ir nuriedėjimas nuo bėgių. Dėl rato pažaidų bei rato ir bėgio kontakte susidarančių smūgio jėgų ratas gali prarasti kontaktą su bėgiu, riedmuo nuriedėti nuo bėgių, sukelti avariją ar katastrofą, o tai – žala aplinkai, žmonių sveikatai ir gyvybei.

Dažnai matematiniuose rato su bėgiu sąveikos modeliuose neįvertinami rato ir bėgio kontakto praradimai, galimi tarpeliai tarp bėgio ir pabėgio bei bėgio ir balasto arba įvertinama tik dalis šių sąlygų.

Tikslinga sukurti rato ir bėgio sąveikos matematinį modelį, leidžiantį tiksliau aprašyti rato iščiuožą ir rato geometriją, įvertinus galimus rato ir bėgio kontakto praradimus ir tarpelius tarp bėgio ir pabėgio bei pabėgio ir balasto.

Dėl rato ir bėgio sąveikos maksimalioms smūgio jėgoms nustatyti kuriami sudėtingi matematiniai modeliai ir kompiuterinės programos. Trūksta mokslinių sprendimų apie supaprastintus maksimalios jėgos skaičiavimo metodus, esant geležinkelio riedmenų ratui su iščiuoža.

**Darbo aktualumas.** Dėl ratų ir bėgių konstrukcijų, bėgių sandūrų ir suvirinimo jungčių, skirtingų traukinio greičių, stabdymo jėgų, rato blokavimo bei riedmens masių susidaro skirtingų dydžių jėgos tarp rato ir bėgio. Šios jėgos sukelia trintį tarp rato ir bėgio, greitesnį jų darbinių paviršių nusidėvėjimą, taip pat gali sutrumpinti jų eksploataavimo laiką.

Daugelis autorių rato su iščiuoža ir bėgio sąveikos matematiniuose modeliuose rato ir bėgio kontakto zoną aprašo tašku, o rato geometriją – analitine funkcija. Nėra sukurto vieno metodo, pagal kurį galima tiksliai nustatyti rato su iščiuoža ir bėgio kontakto zonoje vykstančius dinامينius procesus.

Saugiam traukinių eismui, aplinkai, žmonių sveikatai ir net gyvybei išsaugoti riedmens rato su pažaida ir bėgio sąveika yra aktuali šios srities problema. Tikslinga sukurti supaprastintą metodiką nustatyti rato ir bėgio kontakte vykstančius procesų parametrus, vagonų ir bėgių ilgesniam eksploataavimo laikui užtikrinti.

**Tyrimų objektas** – riedmenų rato su pažaidomis ir bėgio su nelygumais sąveika.

**Darbo tikslas** – sukurti sistemos „Riedmuo-Kelias“ skaičiavimo metodą, leidžiantį nustatyti dėl rato su pažaidomis ir bėgio su nelygumais sąveikos veikiančias dinamines apkrovas, jų priklausomybes nuo šios sistemos geometrinių parametrų ir riedmenų judėjimo greičio.

**Darbo uždaviniai.** Darbo tikslui pasiekti suformuluoti šie uždaviniai:

1. Atlikti riedmens rato ir bėgio sąveikos tyrimų metodų apžvalga ir rato bei bėgio pažaidų analizė.
2. Sukurti sistemos „Riedmuo-Kelias“ matematinį modelį leidžiantį įvertinti rato pažaidas ir bėgio nelygumus ir susidarančius tarpelius tarp rato ir bėgio.
3. Sukurti supaprastintą metodiką leidžiančią įvertinti iščiuožos ilgio ir važiavimo greičio įtaką veikiančioms maksimalioms rato ir bėgio sąlyčio zonoje.
4. Tyrimų rezultatus, gautus pagal sukurtą vertikaliųjų kontaktinių apkrovų nustatymo metodiką, palyginti su kitų autorių tyrimų rezultatais ir realiais eksploatacijos sąlygomis gaunamais sistemos ATLAS-LG duomenimis.

**Tyrimų metodika.** Darbe taikomi tyrimai, pagrįsti Lietuvos ir užsienio šalių mokslininkų šios srities darbuose pateikiamomis metodikomis. Sistemos „Riedmuo-Kelias“ modeliui ir supaprastintai vertikaliųjų kontaktinių jėgų nustatymo metodikai sukurti taikyti analitiniai, skaitiniai (2, 3, 4 skyriuose), statistiniai (4 skyriuje) tyrimų metodai.

Rato ir bėgio kontakto jėgoms nustatyti naudota sistema ATLAS-LG. Bėgių pažaidų analizei buvo pritaikyta specialioji defektoskopo RDM-22s programa.

Matematinis modeliavimas buvo atliekamas „Visual Studio aplinkoje“. Rezultatams apdoroti naudotos „Fortran“, „Maple“, „Matlab“, „R“, „Microsoft Office“ programinės įrangos.

**Darbo mokslinis naujumas**

1. Sukurtas statistiniais, analitiniais ir skaitiniais metodais pagrįstas sistemos „Riedmuo-Kelias“ rato ir bėgio sąveikos matematinis modelis, tinkantis rato su pažaida ir bėgio kontakte vykstantiems dinaminiais procesams įvertinti.
2. Taikant skaitinius ir eksperimentinius metodus, sukurta supaprastinta vertikaliųjų kontaktinių apkrovų nustatymo metodika dėl rato su pažaida ir bėgio sąveikos susidariusioms jėgoms ir kontakto charakteristikoms aprašyti. Taikant supaprastintą kontaktinių apkrovų nustatymo metodiką, nustatoma kontakte veikiančios jėgos esant skirtingiems parametrams.

3. „Visual Studio“ aplinkoje sukurtas sistemos „Riedmuo-Kelias“ modelis, leidžiantis kiekvienu laiko momentu nustatyti: slydimo greičius, išilginius tampruosius slydimus, normalines ir trinties jėgas, trinties jėgų sukimo momentus. Panaudojus šiuos duomenis, kiekvienu laiko momentu rato ir bėgio kontakto zonoje galima nustatyti vidutines šių parametų reikšmes.

***Darbo rezultatų praktinė reikšmė.*** Pateikiamas matematinis sistemos „Riedmuo-Kelias“ riedmenų ratams su pažaida modelis, skirtas dėl rato su pažaida ir bėgio sąveikos veikiančioms jėgoms nustatyti. Modelyje įvertinamas riedmens greitis, sąveikaujančių kūnų geometriniai parametrai, jų fizinės ir mechaninės savybės. Sudarytas matematinis modelis leidžia nustatyti kontakte veikiančių jėgų kitimą.

Naudojant sukurtą matematinę sistemos „Riedmuo-Kelias“ modelį rato ir bėgio kontakto zonoje tarp dviejų kontaktuojančių paviršiaus taškų kiekvienu laiko momentu nustatomi: aširačių tamprusis slydimas, trinties jėgos, trinties jėgų sukimo momentai, veikiantys aširatį, išskirstyta apkrova, kontaktinės jėgos, siekiant pailginti rato ir bėgio eksploatavimo laikotarpį, sumažinti traukinių avaringumą, saugoti aplinką, žmonių sveikatą ir gyvybę.

Vertikaliųjų kontaktinių apkrovų nustatymo metodika skirta tyrinėti rato ir bėgio pažaidų geometrinių parametų poveikį dinaminiams sistemos „Riedmuo-Kelias“ procesams. Panaudojus ATLAS-LG posistemio gautas maksimalių jėgų reikšmes ir supaprastinta vertikaliųjų kontaktinių jėgų nustatymo metodika gautas reikšmes, galima nustatyti ratų pažaidų dydžius.

#### ***Ginamieji teiginiai***

1. Taikant sukurtą sistemos „Riedmuo-Kelias“ matematinę modelį dinaminiams procesams tirti, turi būti įvertinami rato ir bėgio bei jų pažaidų geometriniai parametrai ir susidarantys tarpeliai tarp rato ir bėgio riedėjimo paviršių bei tarp bėgio ir pabėgio.
2. Kiekvienu laiko momentu norint nustatyti kontaktines jėgas ir kitus sistemos „Riedmuo-Kelias“ dinامينius parametrus: slydimo greičius, išilginius tampruosius slydimus, normalines ir trinties jėgas, trinties jėgų sukimo momentus, veikiančius aširatį bei tam kad įvertinti rato ir bėgio pažaidas ar mikronelygumus, rato su bėgiu kontakto zona suskaidomas į tam tikrą skaičių intervalų.
3. Sukūrus supaprastintą vertikaliųjų kontaktinių apkrovų nustatymo metodiką, galima nustatyti dėl rato su pažaida ir bėgio sąveikos veikiančias maksimalias kontaktines jėgas, įvertinant greitį, rato ir bėgio, rato pažaidos geometrinius parametrus ir deviacijos kampą.
4. Panaudojus išvestas kontaktinės smūgio jėgos priklausomybes, galima nustatyti smūgio jėgos pasiskirstymą ir jos dydžio kitimą laike.

**Disertacijos struktūra.** Disertaciją sudaro įvadas, keturi skyriai ir išvados. Darbo apimtis yra 116 puslapių, neskaitant priedų, tekste panaudotos 123 numeruotos formulės, 52 paveikslai ir 9 lentelės. Disertacijoje panaudoti 108 literatūros šaltiniai.

Pirmame skyriuje išanalizuoti geležinkelio bėgių ir aširačių ratų defektai ir pažeidimai, jų atsiradimo priežastys, reikalavimai ir galimi šalinimo būdai.

Antrame skyriuje aprašomas sukurtas sistemos „Riedmuo-Kelias“ matematinis modelis, leidžiantis tirti rato su iščiuožomis ir bėgio sąveiką

Trečiame skyriuje remiantis teorinių ir eksperimentinių tyrimų rezultatais, sukurta ir aprašyta vertikalųjų kontaktinių apkrovų nustatymo metodika (KAM).

Ketvirtame skyriuje pateikiamas pagal sukurtą matematinį modelį atliktas sistemos „Riedmuo-Kelias“, kai ratas turi iščiuožą, dinaminių procesų tyrimas. Supaprastinta vertikalųjų kontaktinių apkrovų nustatymo metodika palyginta su sistemos ATLAS-LG ir kitų autorių kontaktinių jėgų metodais gautomis reikšmėmis.

### ***Bendrosios išvados***

1. Sukurtas sistemos „Riedmuo-Kelias“ matematinis modelis, pagrįstas kontakto zonos padalinimu, įvertinus rato pažeidimus, bėgio nelygumus skverbti. Matematinis modelis tarp dviejų kontaktuojančių paviršiaus taškų kiekvienu laiko momentu leidžia nustatyti tampraus slydimo greičius, išilginius santykinius slydimus, normalines ir trinties jėgas, trinties jėgų sukimo momentus veikiančius aširatį ir vidutines šių charakteristikų reikšmes, siekiant pailginti rato ir bėgio eksploatavimo laikotarpį, sumažinti traukinių eismo avaringumą, saugoti aplinką, žmonių sveikatą ir gyvybę.
2. Naudojant sukurtą sistemos „Riedmuo-Kelias“ matematinį modelį iš-tirti sistemos „Riedmuo-Kelias“ dinaminiai procesai, kai riedmens greitis 100 km/h, statinė ašinė rato apkrova yra 100 kN,  $R_{W0} = 0,495$  m,  $\Delta_F = 2,53$  mm,  $L_F = 100$  mm. Nustatyta, kad smūgio jėga bėgį apkrauna po keturis pabėgius į abi puses nuo labiausiai apkrauto pabėgio. Lyginant rato su iščiuoža ir bėgio vertikalius poslinkius, stebimi trumpalaikiai kontakto praradimai. Gauti sistemos „Riedmuo-Kelias“ vertikalios rato su iščiuoža ir bėgio kontaktinės jėgos bei aširačio pagreičio spektrai. Maksimali vertikalios rato ir bėgio jėgos amplitudė (40 kN) yra esant dažniui  $f = 503$  Hz, pagreičio amplitudės maksimali reikšmė - esant  $2f = 1006$  Hz.
3. Atliktas sistemos „Riedmuo-Kelias“ dinaminių procesų tyrimas, esant skirtingiems važiavimo greičiams prie skirtingų rato iščiuožų dydžių

ir rato statinių apkrovų. Nustatyta, kad kuo didesnis riedmens judėjimo greitis ir mažesnė rato vertikalioji apkrova, masės m<sub>bg1</sub> poslinkių amplitudė bei poslinkio kitimo dažnis yra didesni. Taip pat nustatyta, kad mažėjant rato spinduliui ir didėjant deviacijos kampui smūgio jėga didėja, o didėjant proporcingumo koeficientui, maksimali smūgio jėga mažėja.

4. Supaprastinta vertikalinių kontaktinių apkrovų nustatymo metodika palyginta su sistemos ATLAS-LG ir kitų autorių kontaktinių jėgų metodais gautomis reikšmėmis. Nustatyta, kad siūloma metodika sutampa su kitų autorių metodais gautomis reikšmėmis, kai judėjimo greitis iki 56 km/h. Esant greičiams iki 56 km/h su sistema ATLAS-LG tikrinamos traukinių kontaktinės jėgos, veikiančios bėgi.
5. Atlikus statistinių duomenų analizę nustatyta, kad pagal posistemio ATLAS-LG ir supaprastintą vertikalinių kontaktinių apkrovų nustatymo metodiką gautus rezultatus, dispersijos ir santykiniai nuokrypiai skiriasi nereikšmingai ir kad abi skaičiavimo metodikos vienodai patikimos.

### **Trumpos žinios apie autorių**

Rasa Žygienė (Musteikytė) gimė 1973 m. sausio 3 d. Kaune. 1991 m. baigė Kauno 4-ąją vidurinę mokyklą (dabartinę Kauno S. Dariaus ir S. Girėno gimnaziją). 2002 m. baigė Vilniaus Petro Vileišio aukštesniąją transporto mokyklą (dabartinę Vilniaus technologijų ir dizaino kolegiją), suteikta geležinkelio eismo administratoriaus kvalifikacija. 2007 m. Vilniaus Gedimino technikos universitete, suteikta transporto inžinieriaus kvalifikacija. 2009 m. Vilniaus Gedimino technikos universitete suteiktas transporto inžinerijos magistro laipsnis. Nuo 2009 iki 2015 m. Vilniaus Gedimino technikos universiteto transporto inžinerijos mokslų krypties doktorantė. Nuo 1999 iki 2013 m. dirbo AB „Lietuvos geležinkeliai“ filiale „Kauno geležinkelių infrastruktūra“.